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FUZZY α-PREIRRESOLUTE FUNCTIONS

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Invited paper to celebrate Professor Constantin Udrişte,
on the occasion of his seventies

Abstract. In this paper, we introduce and characterize α-preirresolute functions between fuzzy topological spaces and also study these functions in relation to some other types of already known fuzzy functions. We speculate that fuzzy α-preirresolute functions may be relevant to the physics of fractal and cantorian spacetime.

1. INTRODUCTION AND PRELIMINARIES

The concept fuzzy has invaded almost all branches of mathematics with the introduction of fuzzy sets by Zadeh [21] of 1965. The theory of fuzzy topological spaces was introduced and developed by Chang [6] and since then various notions in classical topology have been extended to fuzzy topological spaces. In 1994 Prasad et al. [13] introduced and investigated the notion of fuzzy α-irresolute mappings. Saraf et. al. [15] introduced the notion of fuzzy α-precontinuity. Recently Professor El-Naschie has been shown in [7], [8] and [10] that the notion of fuzzy topology may be relevant to quantum particle physics in connection with string theory and ε^{∞} theory. Thus our motivation in this paper is to define fuzzy α-preirresolute functions and give several characterizations and their properties. We also study these functions comparing with other types of already known functions. This function is stronger than fuzzy preirresolute functions [12] and is a generalization of fuzzy completely weakly preirresolute functions, introduced by Hakeim et.al. [11]. Here also we give some characterizations of fuzzy completely weakly preirresolute functions.

Throughout this paper (X,τ), (Y,σ) and (Z,γ) (or simply X, Y and Z) represent non-empty fuzzy topological spaces on which no separation axioms are assumed, unless otherwise mentioned. The fuzzy set A of X is called fuzzy α-open [3] (resp. fuzzy preopen [3]) if $A \leq \text{Int}(\text{Cl}(\text{Int}(A))$ (resp. $A \leq \text{Int}(\text{Cl}(A)))$, where $\text{Cl}(A)$ and $\text{Int}(A)$ denote the closure of A and the interior of A respectively. The fuzzy subset B of X is said to be fuzzy α-closed (resp. preclosed) if, its complement B^c is

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fuzzy α-open (resp. is fuzzy preopen) in X. By $F\alpha O(X)$ (resp. $F\alpha C(X)$, $FPO(X)$, $FPC(X)$), we denote the family of all fuzzy α-open (resp. fuzzy α-closed, fuzzy preopen, fuzzy preclosed) sets of X. The intersection of all fuzzy preclosed sets containing A is called the preclosure of A and is denoted by $\mathrm{p\,Cl}(A)$ [16]. The fuzzy preinterior [16] of A denoted by $\mathrm{p\,Int}(A)$, is defined by the union of all fuzzy preopen sets of X contained in A.

A function $f : (X, \tau) \to (Y, \sigma)$ is said to be:

(i) fuzzy completely weakly preirresolute [11] (resp. preirresolute [12]) if, $f^{-1}(V)$ is fuzzy open (resp. fuzzy preopen) in (X, τ) for every fuzzy preopen set V of (Y, σ);

(ii) fuzzy strongly α-irresolute [18] (resp. fuzzy α-irresolute [13], fuzzy α-precontinuous [15]) if, $f^{-1}(V)$ is fuzzy open (resp. fuzzy α-open, preopen) in (X, τ) for every fuzzy α-open set V of (Y, σ);

(iii) fuzzy α-continuous [17] if, $f^{-1}(V)$ is fuzzy α-open in (X, τ) for every fuzzy open set V of (Y, σ).

A fuzzy point x_p is said to be quasi-coincident with a fuzzy set A in X denoted by $x_p qA$ if $p + A(x) > 1$. Two fuzzy sets A and B are said to be quasi-coincident (q-coincident, shorty) denoted by AqB, if there exists $x \in X$ such that $A(x)+B(x) > 1$ [14].

Lemma 1.1. [20] *Let $f\colon (X, \tau) \to (Y, \sigma)$ be a function and x_p be a fuzzy point of X. Then:*

(1) *$f(x_p)qB \Rightarrow x_p qf^{-1}(B)$, for every fuzzy set B of Y;*

(2) *$x_p qA \Rightarrow f(x_p)qf(A)$, for every fuzzy set A of X.*

A fuzzy set A is said to be pQ-neighborhood (pQ-nbd) of x_p if there is a fuzzy preopen set H such that $x_p qH$ and $H \leq A$, where we consider $H \leq A$ if $H(x) \leq A(x)$ for all $x \in X$.

2. FUZZY α-PREIRRESOLUTE FUNCTIONS

First we discuss some characterization of fuzzy completely weakly preirresolute functions.

Theorem 2.1. *A function $f\colon (X, \tau) \to (Y, \sigma)$ is fuzzy completely weakly preirresolute if and only if for every fuzzy set A of Y $f^{-1}(\mathrm{p\,Int}(A)) \leq \mathrm{Int}(f^{-1}(A))$.*

Proof. Let A be a fuzzy set of Y. Since $\mathrm{p\,Int}(A)$ is fuzzy preopen $f^{-1}(\mathrm{p\,Int}(A))$ is fuzzy open X. We have $f^{-1}(\mathrm{p\,Int}(A)) = \mathrm{Int}(f^{-1}(\mathrm{p\,Int}(A))) \leq \mathrm{Int}(f^{-1}(A))$.

Conversely. Let A be any preopen set in Y, then $\mathrm{p\,Int}(A) = A$ and

$$f^{-1}(A) = f^{-1}(\mathrm{p\,Int}(A)) \leq \mathrm{Int}(f^{-1}(A)),$$

this show that $f^{-1}(A)$ is fuzzy open in X. □

Theorem 2.2. *A function $f\colon (X, \tau) \to (Y, \sigma)$ be one-to-one and onto, then f is fuzzy completely weakly preirresolute if and only if for every fuzzy set A of X, $\mathrm{p\,Int}(f(A)) \leq f(\mathrm{Int}(A))$.*

Proof. Let A be a fuzzy set of X. Since $\mathrm{p\,Int}(f(A)) \in FPO(Y)$, $f^{-1}(\mathrm{p\,Int}(f(A)))$ is fuzzy open in X. Now f is one-to-one and onto so $f^{-1}(f(A)) = A$. Therefore by Theorem 2.1, $f^{-1}(\mathrm{p\,Int}(f(A))) \leq \mathrm{Int}(f^{-1}(f(A))) = \mathrm{Int}(A)$. Hence $f(f^{-1}(\mathrm{p\,Int}(f(A)))) \leq f(\mathrm{Int}(A))$. Since f is onto $\mathrm{p\,Int}(f(A)) = f(f^{-1}(\mathrm{p\,Int}(f(A)))) \leq f(\mathrm{Int}(A))$.

Conversely. Let H be a fuzzy preopen set of Y. Then $H = \mathrm{p\,Int}(H)$. By hypothesis, $f(\mathrm{Int}(f^{-1}(H))) \geq +\mathrm{p\,Int}(f(f^{-1}(H))) = \mathrm{p\,Int}(H) = H$. Hence

$$f^{-1}(f(\mathrm{Int}(f^{-1}(H)))) \geq f^{-1}(H).$$

Since f is one-to-one, we have $\mathrm{Int}(f^{-1}(H)) \geq f^{-1}(H)$. Therefore $f^{-1}(H)$ is fuzzy open in X. □

Theorem 2.3. *Let (x, τ) and (Y, σ) be fuzzy topological space and $f\colon (X, \tau) \to (Y, \sigma)$. Then the following conditions are equivalent:*

(1) *f is fuzzy completely weakly preirresolute;*

(2) *for every fuzzy point x_p in X and $B \in FPO(Y)$ such that $f(x_p)qB$, there is a fuzzy open set A in X such that that x_pqA and $f(A) \leq B$;*

(3) *for every fuzzy point x_p in X and every* pQ*-neighborhood B of $f(x_p)$, $f^{-1}(B)$ is* pQ*-neighborhood of x_p;*

(4) *for every fuzzy point x_p in X and every* pQ*-neighborhood B of $f(x_p)$, there is a* pQ*-neighborhood H of x_p, such that $f(H) \leq B$.*

Proof. (1) $\Rightarrow$ (2): Let x_p be a fuzzy point of X and $B \in FPO(Y)$ such that $f(x_p)qB$. Let $A = f^{-1}(B)$. Then A is fuzzy open in X, such that x_pqA, and $f(A) = f(f^{-1}(B)) \leq B$.

(2) $\Rightarrow$ (3): Let x_p be a fuzzy point of X and B be pQ-neighborhood of $f(x_p)$. Then there exists $W \in FPO(Y)$ such that $f(x_p)qW \leq B$. By hypothesis there is a fuzzy open set A in X, such that $f(x_p)qA$ and $f(A) \leq W$. Thus $x_pqA \leq f^{-1}(W) \leq f^{-1}(B)$. Hence $f^{-1}(B)$ is pQ-neighborhood of x_p.

(3) $\Rightarrow$ (4): Let x_p be a fuzzy point of X and B be pQ-neighborhood of $f(x_p)$. Then $H = f^{-1}(B)$ is pQ-neighborhood of x_p and $f(H) = f(f^{-1}(B)) \leq B$.

(4) $\Rightarrow$ (2): Let x_p be a fuzzy point of X and $B \in FPO(Y)$ such that $f(x_p)qB$ is a pQ-neighborhood of $f(x_p)$. So there is a pQ-neighborhood H of x_p, such that $f(H) \leq B$. Therefore there exist a fuzzy open set A in X such that $x_pqA \leq H$. Hence x_pqA and $f(A) \leq f(H) \leq B$.

(2) $\Rightarrow$ (1): Let $B \in FPO(Y)$ and x_p be a fuzzy point of $f^{-1}(B)$. Clearly $f(x_p) \in B$. Choose the fuzzy point x_p^c as $x_p^c(x) = 1 - x_p(x)$. Then $f(x_p^c)qB$. And so by (2), there exists a fuzzy open set A such that x_p^cqA and $f(A) \leq B$. Now $x_p^cqA \Rightarrow x_p^c(x) + A(X) = 1 - x_p(x) + A(x) > 1 \Rightarrow A(x) > x_p(x) \Rightarrow x_p \in A$. Thus $x_p \in A \leq f^{-1}(B)$. Hence $f^{-1}(B)$ is fuzzy open in X. □

Definition 2.1. A function $f\colon (X, \tau) \to (Y, \sigma)$ is said to be fuzzy α-preirresolute if, $f^{-1}(V)$ is fuzzy α-open in (X, τ), for every fuzzy preopen set V of (Y, σ).

From the definitions stated, we have the following diagram:

F.c.w-preirresoluteness	$\Rightarrow$	*F.α-preirresoluteness*	$\Rightarrow$	*F.preirresoluteness*
$\Downarrow$		$\Downarrow$		$\Downarrow$
s.F.α-irresoluteness	$\Rightarrow$	*F.α-preirresoluteness*	$\Rightarrow$	*F.α-precontinuity*

where:

$F.c.w$-preirresoluteness = Fuzzy completely weakly preirresoluteness;
$F.\alpha$-preirresoluteness = Fuzzy α-preirresoluteness;
F.preirresoluteness = Fuzzy preirresoluteness;
$s.F.\alpha$-irresoluteness = strongly Fuzzy α-irresoluteness;
$F.\alpha$-preirresoluteness = Fuzzy α-preirresoluteness;
$F.\alpha$-precontinuity = Fuzzy α-precontinuity.

Remark 2.1. However, converses of the above implications are not true in general, by [15, 18, 13, 11] and the following examples.

Example 2.1. Let $X = \{a, b\}$ and $Y = \{x, y\}$. Define fuzzy sets $A(a) = 0.4$, $A(b) = 0.4$; $B(x) = 0.3$, $B(y) = 0.4$. Let $\tau = \{0, A, 1\}$ and $\sigma = \{0, B, 1\}$. Then the map $f: (X, \tau) \to (Y, \sigma)$ defined by $f(a) = x$, $f(b) = y$ is fuzzy preirresolute but not fuzzy α-preirresolute, because $Z(x) = 0.8$, $Z(y) = 0.8$ are fuzzy preopen in Y, but $f^{-1}(Z)$ is not fuzzy α-open in X.

Example 2.2. Let $X = \{a, b\}$ and $Y = \{x, y\}$. Define fuzzy sets $A(a) = 0.6$, $A(b) = 0.5$; $B(a) = 0$, $B(b) = 0.8$; $H(x) = 0.5$, $H(y) = 0.5$; $E(x) = 0.7$, $E(y) = 0.8$. Let $\tau = \{0, A, 1\}$, $\Gamma = \{0, B, 1\}$; $\sigma = \{0, H, 1\}$ and $\upsilon = \{0, E, 1\}$.

(i) The mapping $f: (X, \tau) \to (Y, \sigma)$ defined by $f(a) = x$, $f(b) = y$ is fuzzy α-preirresolute but not fuzzy completely weakly preirresolute, because $Z(x) = 0.7$, $Z(y) = 0.7$ are fuzzy preopen in (Y, σ) but $f^{-1}(Z)$ is not fuzzy open in X.

(ii) The mapping $g: (X, \Gamma) \to (Y, \upsilon)$ defined by $g(a) = x$, $g(b) = y$ is fuzzy α-irresolute but not fuzzy α-preirresolute, because $M(x) = 0.4$, $M(y) = 0.5$ are fuzzy preopen in (Y, υ) but $g^{-1}(M)$ is not fuzzy α-open in X.

Theorem 2.4. *For a function $f: (X, \tau) \to (Y, \sigma)$, the following are equivalent:*

(1) *f is fuzzy α-preirresolute;*

(2) *for every fuzzy point x_t in X and every fuzzy preopen set V of Y containing $f(x_t)$, there exist a fuzzy α-open set U of X containing x_t such that $f(U) \leq V$;*

(3) *for every fuzzy point x_t in X and every fuzzy preopen set V of Y containing $f(x_t)$, there exists a fuzzy α-open set U in X containing x_t such that $x_t \in U \leq f^{-1}(V)$;*

(4) *for every fuzzy point x_t in X, the inverse image of each preneighborhood of $f(x_t)$ is an α-neighborhood* [13] *of x_t;*

(5) *for every fuzzy point x_t in X and each preneighborhood B of $f(x_t)$, there exists an α-neighborhood A of x_t such that $f(A) \leq B$;*

(6) $f^{-1}(V) \leq \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(f^{-1}(V))))$ *for every fuzzy preclosed set V of Y;*

(7) *$f^{-1}(H)$ is fuzzy α-closed in X for every fuzzy preclosed set H of Y;*

(8) $\mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(f^{-1}(B)))) \leq f^{-1}\mathrm{p}\,\mathrm{Cl}(B))$ *for every fuzzy subset B of Y;*

(9) $f(\mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(A)))) \leq \mathrm{p\,Cl}(f(A))$ *for every fuzzy subset A of X.*

Proof. (1) ⇔ (2) ⇔ (3); (4) ⇒ (5): Obvious

(2) ⇒ (6): Let V be any fuzzy preopen set of Y and $x_t \in f^{-1}(V)$. By (2), there exists a fuzzy α-open set U of X containing x_t such that $f(U) \leq V$. Thus we have $x_t \in U \leq \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(U))) \leq \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(f^{-1}(V))))$ and hence

$$f^{-1}(V) \leq \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(f^{-1}(V))).$$

(6) ⇒ (7): Let H be any fuzzy preclosed set of Y. Set $V = Y - H$, then V is fuzzy preopen in Y. By (6) we obtain $f^{-1}(V) \leq \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(f^{-1}(V)))$ and hence

$$f^{-1}(H) = X - f^{-1}(Y - H) = X - f^{-1}(V)$$

is fuzzy α-closed in X.

(7) ⇒ (8): Let B be any fuzzy set of Y. Since $\mathrm{p\,Cl}(B)$ is a fuzzy preclosed subset of Y, then $f^{-1}(\mathrm{p\,Cl}(B))$ is fuzzy α-closed in X and hence

$$\mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(f^{-1}(\mathrm{p\,Cl}(B))))) \leq f^{-1}\mathrm{p\,Cl}(B)).$$

Therefore we obtain $\mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(f^{-1}(B)))) \leq f^{-1}\mathrm{p\,Cl}(B))$.

(8) ⇒ (9): Let A be any fuzzy set of X. by (8), we have

$$\mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(A)) \leq \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(f^{-1}(f(A)))) \leq f^{-1}(\mathrm{p\,Cl}(f(A)))$$

and hence $f(\mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(A)))) \leq \mathrm{p\,Cl}(f(A))$.

(9) ⇒ (1): Let V be any fuzzy preopen set of Y. Since $f^{-1}(Y-V) = X - f^{-1}(V)$ is a fuzzy set of X and by (9), we obtain

$$f(\mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(f^{-1}(Y-V)))) \leq \mathrm{p\,Cl}(f(f^{-1}(Y-V))) \leq \mathrm{p\,Cl}(Y-V) = Y - \mathrm{p\,Int}(V) = Y - V$$

and hence

$$\begin{aligned} X - \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(f^{-1}(V)))) &= \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(X - f^{-1}(Y)))) = \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(f^{-1}(Y - V))) \\ &\leq f^{-1}(f(\mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(f^{-1}(Y - V)))))) \leq f^{-1}(Y - V) \\ &= X - f^{-1}(V). \end{aligned}$$

Therefore, we have $f^{-1}(V) \leq \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(f^{-1}(V))))$ and hence $f^{-1}(V)$ is fuzzy α-open in X. Thus, f is fuzzy α-preirresolute.

(1) ⇒ (4): Let x_t be a fuzzy point in X and V be any preneighbourhood of $f(x_t)$, then there exists a fuzzy preopen set G of Y such that, $f(x_t) \in G \leq V$. Now $f^{-1}(G)$ is fuzzy α-open in X and $x_t \in f^{-1}(G) \leq f^{-1}(V)$. Thus $f^{-1}(V)$ is an α-neighbourhood of x_t in X.

(5) ⇒ (2): Let x_t be a fuzzy point in X and V is fuzzy preopen set of Y such that $f(x_t) \in V$. Then V is preneighbourhood of $f(x_t)$, so there is an α-neighbourhood A of x_t such that $x_t \in A$, and $f(A) \leq V$. Hence there exists a fuzzy α-open set U in X such that $x_t \in U \leq A$, and so $f(U) \leq f(A) \leq V$. □

Theorem 2.5. *For a function $f\colon (X,\tau) \to (Y,\sigma)$, the following are equivalent:*

(1) *f is fuzzy α-preirresolute;*

(2) *for each point* x_t *of* X *and every fuzzy preopen set* B *of* Y *such that* $f(x_t)qB$, *there exists a fuzzy* α*-open set* A *of* X *such that* x_tqA *and* $f(A) \leq B$;

(3) *for every fuzzy point* x_t *in* X *and every fuzzy preopen set* B *of* Y *such that* $f(x_t)qB$, *there exists a fuzzy* α*-open set* A *of* X *such that* x_tqA *and* $A \leq f^{-1}(B)$.

Proof. (1) $\Rightarrow$ (2) Let x_t be a fuzzy point in X and B be a fuzzy preopen set of Y such that $f(x_t)qB$. Then $f^{-1}(B)$ is fuzzy α-open in X, and $x_tqf^{-1}(B)$ by Lemma 1.1. If we take $A = f^{-1}(B)$ then x_tqA and $f(A) = f(f^{-1}(B)) \leq B$.

(2) $\Rightarrow$ (3) Let x_t be a fuzzy point in X and B be a fuzzy preopen set of Y such that $f(x_t)qB$. Then by (2), there exists a fuzzy α-open set A of X such that x_tqA and $f(A) \leq B$. Hence we have x_tqA and $A \leq f(f^{-1}(A)) \leq f^{-1}(B)$.

(3) $\Rightarrow$ (1) Let B be a fuzzy preopen set of Y and x_t be a fuzzy point in X such that $x_t \in f^{-1}(B)$. Then $f(x_t) \in B$. Choose the fuzzy point $x_t^c(x) = 1 - x_t(x)$. Then $f(x_t^c)qB$. And so by (3), there exists a fuzzy α-open set A of X such that x_t^cqA and $f(A) \leq B$. Now x_t^cqA implies $x_t^c(x) + A(X) = 1 - x_t(x) + A(x) > 1$. It follows that $x_t \in A$. Thus $x_t \in A \leq f^{-1}(B)$. Hence $f^{-1}(B)$ is fuzzy α-open in X. □

Theorem 2.6. *A function* $f\colon (X,\tau) \to (Y,\sigma)$ *is fuzzy* α*-preirresolute if the graph function* $g\colon (X,\tau) \to (X \times Y, \tau \times \sigma)$, *is fuzzy* α*-preirresolute.*

Proof. Let A be a fuzzy preopen set of Y, then $1_x \times A$ is fuzzy preopen in $X \times Y$. Since g is fuzzy α-preirresolute $g^{-1}(1_x \times A) \in F\alpha O(X)$. But by [[1], Lemma 2.4] $f^{-1}(A) = 1_x \cap f^{-1}(A) = g^{-1}(1_x \times A)$. Hence f is fuzzy α-preirresolute. □

Theorem 2.7. *If* $f\colon (X,\tau) \to (Y,\sigma)$ *is fuzzy* α*-preirresolute and* A *is fuzzy preopen subset of* X, *then the restriction* $f \mid A\colon A \to Y$ *is fuzzy* α*-preirresolute.*

Proof. Let V be any fuzzy preopen set of Y. Since f is fuzzy α-preirresolute, $f^{-1}(V)$ is fuzzy α-open in X. Since A is fuzzy preopen in X, then $(f \mid A)^{-1}(V) = A \cap f^{-1}(V)$ is fuzzy α-open in A and hence $f \mid A$ is fuzzy α-preirresolute. □

Theorem 2.8. *Let* $f\colon (X,\tau) \to (Y,\sigma)$ *be a function and* $\{A_i : i \in I\}$ *be a cover of* X *by fuzzy* α*-open sets of* X. *Then* f *is fuzzy* α*-preirresolute if* $f \mid A_i\colon A_i \to Y$ *is fuzzy* α*-preirresolute for each* $i \in I$.

Proof. Let V be any fuzzy preopen set of Y. Since $f \mid A_i$ is fuzzy α-preirresolute. Then $(f \mid A_i)^{-1}(V)$ is fuzzy α-open in A_i and since $A_i \in F\alpha O(X)$, then $f \mid A_i)^{-1}(V)$ is fuzzy α-open in X, for each $i \in I$. Therefore

$$f^{-1}(V) = X \cap f^{-1}(V) = \cup\{A_i \cap f^{-1}(V) : i \in I\} = \cup\{(f \mid A_i)^{-1}(V) : i \in I\}$$

is fuzzy α-open in X because the union of fuzzy α-open sets is fuzzy α-open set. Hence f is fuzzy α-preirresolute. □

Theorem 2.9. *If* $f\colon X \to X_1 \times X_2$ *is fuzzy* α*-preirresolute, then* $P_i \circ f\colon X \to X_i$ *is fuzzy* α*-preirresolute, where* $P_i\colon X_1 \times X_2 \to X_i$ $(i = 1, 2)$ *is the projection of* $X_1 \times X_2$ *onto* X_1.

Proof. Let $V_i \in FPO(X_i)$. Since P_i is continuous and open [19] it is fuzzy preirresolute [[2], proposition 3]. Therefore $P_i^{-1}(V_i)$ is fuzzy preopen in $X_1 \times X_2$. Since f is fuzzy α-preirresolute then $f^{-1}(P_i^{-1}(V_i)) = (P_i \circ f)^{-1}(V_i) \in F\alpha O(X)$. Hence $P_i \circ f$ is fuzzy α-preirresolute. □

Theorem 2.10. *Let $f\colon (X,\tau) \to (Y,\sigma)$ and $g\colon (Y,\sigma) \to (Z,\gamma)$ be functions. Then the composition $g \circ f\colon (X,\tau) \to (Z,\gamma)$ is fuzzy α-preirresolute if:*
(i) *f is fuzzy α-preirresolute and g is fuzzy preirresolute;*
(ii) *f is fuzzy α-irresolute and g is fuzzy α-preirresolute;*
(iii) *f is fuzzy α-continuous and g is fuzzy completely weakly preirresolute.*

Proof. (i) Let W be any fuzzy preopen set of Z. Since g is fuzzy preirresolute, $g^{-1}(W)$ is fuzzy preopen in Y. Since f is fuzzy α-preirresolute, then $(g \circ f)^{-1}(W) = f^{-1}(g^{-1}(W))$ is fuzzy α-open in X and hence $g \circ f$ is fuzzy α-preirresolute.
(ii) and (iii): By similar technique of (i). □

3. CONCLUSION

The fundamental definitions and theories of scientific studies specially in mathematical ones, with respect to ordinary sets are considered as a particular case of the corresponding fuzzy notion. It is natural to extend the concept of point set topology to fuzzy sets, resulting in the theory of fuzzy topology, [4], [5]. The concept of fuzzy topology may be relevant to quantium particles physics particulary in connection with string theory and ε^∞ theory [7, 8, 9, 10]. Thus our motivation in this paper is to define fuzzy α-preirresolute functions and give several characterizations and their properties.

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