

SOME ASPECTS OF APPROXIMATIVE G -ATOMIC DECOMPOSITIONS IN BANACH SPACES

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ABSTRACT. In this paper, we introduce and study the approximative G -atomic decomposition ($AGAD$) as a generalization of atomic decomposition in Banach spaces. A sufficient condition for the existence of $AGAD$ has been obtained. In the sequel, some characterizations of $AGAD$ have been given regarding complemented subspaces, bounded approximation property, and Schauder approximative G -atomic decomposition. Further, we investigate relations among various existing notions with $AGAD$. Finally, we deal with applications of $AGAD$ concerning recovery of a signal in a discrete signal space $L^2(\Omega)$ and eigenvalue problems.

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