

**SOME UNIFYING NON-UNIQUE FIXED POINT THEOREMS FOR  
AKRAM TYPE CONTRACTIONS IN  $b$ -METRIC SPACE**

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**ABSTRACT.** In this paper, we prove some unifying fixed point theorems of Ćirić's type in a complete  $b$ -metric space for mappings possessing non-unique fixed points by employing general contractive inequality conditions as well as the notion of comparison functions. The right-hand side of each of the contractive inequality condition being introduced is Akram-Zafar-Siddiqui while the left-hand side of each is Akram-Zafar-Siddiqui type to certain extent. Our results unify, generalize and improve several unique and non-unique fixed point theorems in the literature. Also, the results obtained are extensions of such unique and non-unique fixed point theorems from the metric space setting to the  $b$ -metric space environment.

REFERENCES

- [1] J. Achari: *Results on nonunique fixed points*, Publ. Inst. Math. (Beograd) (N.S.), **26**(1979), No. 40, 5-9.
- [2] J. Achari: *On the generalization of Pachpatte's nonunique fixed point theorem*, Indian J. Pure Appl. Math., **13**(1982), No. 3, 299-302.
- [3] R.P. Agarwal, M. Meehan and D. O'Regan: *Fixed Point Theory and Applications*, Cambridge University Press, 2004.
- [4] M. Akram, A.A. Zafar, A.A. Siddiqui: *A general class of contractions: A-contractions*, Novi Sad J. Math., **38**(2008), No. 1, 25-33.
- [5] S. Banach: *Sur les operations dans les ensembles abstraits et leur applications aux equations integrales*, Fund. Math., **3**(1922), 133-181.
- [6] V. Berinde: *Approximating fixed points of weak contractions using Picard iteration*, Nonlinear Anal. Forum, **9**(2004), No. 1, 43-53.
- [7] V. Berinde: *Iterative Approximation of Fixed Points*, Springer-Verlag, 2007.
- [8] S.K. Chatterjea: *Fixed-Point Theorems*, C. R. Acad. Bulgare Sci., **10**(1972), 727-730.
- [9] Lj.B. Ćirić: *On contraction type mappings*, Math. Balkanica (N.S.), **1**(1971), 52-57.
- [10] Lj.B. Ćirić: *On some maps with a nonunique fixed point*, Publ. Inst. Math. (Beograd) (N.S.), **17**(1974), No.31, 52-58.
- [11] Lj.B. Ćirić and N. Jotić: *A further extension of maps with nonunique fixed points*, Mat. Vesnik, **50**(1998), 1-4.
- [12] Lj. B. Ćirić: *Some recent results in metrical fixed point theory*, University of Belgrade, 2003.
- [13] S. Czerwik: *Nonlinear set-valued contraction mappings in  $b$ -metric spaces*, Atti Sem. Mat. Fis. Univ. Modena, **46**(1998), No. 2, 263-276.
- [14] B.C. Dhage: *Some results for the maps with a nonunique fixed point*, Indian J. Pure Appl. Math., **16**(1985), No. 3, 245-256.

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- [15] E. Karapinar: *Some nonunique fixed point theorems of Ćirić type on cone metric spaces*, Abstract and Applied Analysis, **2010**(2010), Article ID 123094, 14 pages.
- [16] D.S. Jaggi: *Some unique fixed point theorems*, Indian J. Pure Appl. Math., **8**(1977), No. 2, 223-230.
- [17] R. Kannan: *Some results on fixed points*, Bull. Calcutta Math. Soc., **10**(1968), 71-76.
- [18] M.A. Khamsi and W.A. Kirk: *An Introduction to Metric Spaces and Fixed Point Theory*, John Wiley & Sons, Inc., 2001.
- [19] A.R. Khan, V. Kumar and N. Hussain: *Analytical and numerical treatment of Jungck-type iterative scheme*, Appl. Math. Comput., **231**(2014), 521-535.
- [20] M.G. Maia: *Un'osservazione sulle contrazioni metriche*, Rend. Semin. Mat. Univ. Padova, **40**(1968), 139-143.
- [21] M.O. Olatinwo: *Some new Fixed point theorems in complete metric spaces*, Creat. Math. Inform., **21**(2012), No. 2, 189-196.
- [22] M.O. Olatinwo: *Some Banach type fixed point theorems and implicit type error estimates*, Kochi J. Math., **8**(2013), 105-117.
- [23] M.O. Olatinwo: *Non-unique fixed point theorems of Ćirić's type for rational hybrid contractions*, J. Nanjing Univ. Nat. Sci., **31**(2014), No. 2, 140-149.
- [24] M.O. Olatinwo: *Some Ćirić's type non-unique fixed point theorems and rational type contractive conditions*, Kochi J. Math., **10**(2015), 1-9.
- [25] M.O. Olatinwo: *Some non-unique fixed point theorems of Ćirić's type using rational type contractive conditions*, Georgian Math. J., **24**(2017), No. 3, 455-461.
- [26] M.O. Olatinwo: *A new generalization of non-unique fixed point theorems of Ćirić for Akram-Zafar-Siddiqui type contraction*, J. Math. Sci. Modelling, **1**(2018), No. 3, 153-157.
- [27] M.O. Olatinwo: *Some generalizations of nonunique fixed point theorems of Ćirić-type for  $(\Phi, \Psi)$ -hybrid contractive mappings*, Ann. Math. Sil., **33**(2019), 221-234.
- [28] M.O. Olatinwo: *Some non-unique common fixed point theorems for Ćirić-Akram-Zafar-Siddiqui hybrid type mappings*, JP J. Fixed Point Theory Appl., **16**(2021), No. 1, 33-65.
- [29] B.G. Pachpatte: *On Ćirić type maps with a nonunique fixed point*, Indian J. Pure Appl. Math., **10**(1979), No. 8, 1039-1043.
- [30] I.A. Rus: *Generalized Contractions and Applications*, Cluj University Press, Cluj Napoca, 2001.
- [31] I.A. Rus, A. Petrusel and G. Petrusel: *Fixed Point Theory 1950-2000: Romanian Contributions*, House of the Book of Science, Cluj Napoca, 2002.
- [32] T. Zamfirescu: *Fix point theorems in metric spaces*, Arch. Math. (Basel), **23**(1972), 292-298.
- [33] E. Zeidler: *Nonlinear Functional Analysis and its Applications-Fixed Point Theorems*, Springer-Verlag, New York, Inc., 1986.

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