

OSCILLATION AND NONOSCILLATION FOR CONFORMABLE FRACTIONAL DIFFERENTIAL EQUATIONS AND INCLUSIONS

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ABSTRACT. In this article, we prove some existence, oscillatory and nonoscillatory results for some classes of conformable fractional differential equations and inclusions. Our results are based on the set-valued analysis, the fixed point theory, and the method of upper and lower solutions.

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