

## APPROXIMATE EXPONENTIALLY BIVARIATIONAL INEQUALITIES

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ABSTRACT. Some new classes of exponentially bivariational inequalities are introduced and considered in this paper. It is shown that the optimality conditions of differentiable exponentially biconvex functions can be characterized by exponentially bivariational inequalities. Several inertial iterative are proposed and analyzed for solving exponentially bivariational inequalities by applying the auxiliary principle technique. Convergence criteria is discussed using the pseudo-monotonicity of the operator. Special cases are also highlighted as applications of the main results. Results obtained in this paper can be viewed as significant improvement of previously known results. It is an interesting problem to implement these methods numerically and their comparison with other techniques.

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