

VISCOSITY APPROXIMATION METHODS FOR FINDING MINIMUM-NORM FIXED POINT OF NONEXPANSIVE MAPPINGS IN HILBERT SPACE

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ABSTRACT. New iterative algorithms are considered for finding minimum-norm fixed point of nonexpansive mappings by implicit and explicit methods. We obtained strong convergence theorems for implicit and explicit methods in real Hilbert spaces. In the sequel, we apply our main results to find solutions of minimizer of a continuously Fréchet-differentiable convex functional and linear inverse problem each of which has the minimum norm in Hilbert spaces. Our main results improve and compliment some known results.

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