

OPTIMIZATION PROBLEMS WITH DIFFERENCE OF SET-VALUED MAPS UNDER HIGHER-ORDER ρ -CONE CONVEXITY

KOUSHIK DAS

ABSTRACT. In this paper, an optimization problem (DP) is studied where the objective maps and the constraints are the difference of set-valued maps (abbreviated as SVMs). Under the higher-order contingent epiderivative and higher-order ρ -cone convexity assumptions, the higher-order sufficient Karush-Kuhn-Tucker (KKT) optimality conditions are established for the problem (DP). As a special case, the results coincide with the existing ones available in the literature.

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*Education Directorate, Department of Higher Education
Government of West Bengal
Bikash Bhawan, Saltlake, Kolkata - 700091, India*

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*Department of Mathematics
Government General Degree College Singur
Hooghly - 712409, West Bengal, India*

Email address: koushikdas.maths@gmail.com; koushik@singurgovtcollege.org