

NEW CLASSES OF EXPONENTIALLY CONVEX FUNCTIONS

KHALIDA INAYAT NOOR AND MUHAMMAD ASLAM NOOR

Dedicated to the memories of our beloved and dedicated visionary parents

ABSTRACT. The main objective of this paper is to introduce a new kind of exponentially convex set and convex functions. We investigate several properties of the exponentially convex functions. Several interesting results characterizing the exponentially convex functions are obtained. Optimality conditions of the differentiable exponentially convex functions are characterized by a class of variational inequalities. Results obtained in this paper can be viewed as significant improvement of previously known results.

REFERENCES

- [1] N.I. Akhiezer: *The Classical Moment Problem and Some Related Questions in Analysis*, Oliver and Boyd, Edinburgh, U. K., 1965.
- [2] G. Alirezaei and R. Mazhar: *On exponentially concave functions and their impact in information theory*, 2018 Information Theory and Applications Workshop (ITA), **9**(2018), No. 5, 265-274.
- [3] T. Antczak: *On (p, r) -invex sets and functions*, J. Math. Anal. Appl., **263**(2001), 355-379.
- [4] M. Avriel: *r-Convex functions*, Math. Program., **2**(1972), 309-323.
- [5] M.U. Awan, M.A. Noor and K.I. Noor: *Hermite-Hadamard inequalities for exponentially convex functions*, Appl. Math. Inf. Sci., **12**(2018), No. 2, 405-409.
- [6] S.N. Bernstein: *Sur les fonctions absolument monotones*, Acta Math., **52**(1929), 1-66.
- [7] G. Cristescu and L. Lupşa: *Non-Connected Convexities and Applications*, Kluwer Academic Publisher, Dordrecht, 2002 (APOP 68).
- [8] S.S. Dragomir: *Inequalities of Hermite-Hadamard type for φ -convex functions*, Konuralp J. Math., **4**(2016), No. 1, 54-67.
- [9] S.S. Dragomir and I. Gomm: *Some Hermite-Hadamard type inequalities for functions whose exponentials are convex*, Stud. Univ. Babeş-Bolyai Math., **60**(2015), No. 4, 527-534.
- [10] S.S. Dragomir and I. Gomm: *Some Hermite-Hadamard type inequalities for functions whose product with exponentials are convex*, Sci. Stud. Res. Ser. Math. Inform., **25**(2015), No. 1, 145-158.
- [11] C.F. Niculescu and L.E. Persson: *Convex Functions and Their Applications*, Springer-Verlag, New York, 2018.
- [12] M.A. Noor: *New approximation schemes for general variational inequalities*, J. Math. Anal. Appl., **251**(2000), 217-229.
- [13] M.A. Noor: *Some developments in general variational inequalities*, Appl. Math. Comput., **152**(2004), 199-277.
- [14] M.A. Noor and K.I. Noor: *Strongly exponentially convex functions*, Politehn. Univ. Bucharest Sci. Bull. Ser. A Appl. Math. Phys., **81**(2019), No. 4, 75-84.
- [15] M.A. Noor and K.I. Noor: *Strongly exponentially convex functions and their properties*, J. Adv. Math. Stud., **12**(2019), No. 2, 177-185.

Received: January 08, 2026. *Revised:* July 28, 2026.

2020 Mathematics Subject Classification: 49J40, 26D15, 26D10, 90C33.

Key words and phrases: Convex functions, minimum, exponentially convex functions, variational inequalities.

- [16] M.A. Noor and K.I. Noor: *On generalized strongly convex functions involving bifunction*, Appl. Math. Inf. Sci., **13**(2019), No. 3, 411-416.
- [17] M.A. Noor and K.I. Noor: *Exponentially general convex functions*, Transyl. J. Math. Mechanics, **11**(2019), No. 1-2, 141-153.
- [18] M.A. Noor and K.I. Noor: *New classes of strongly exponentially preinvex functions*, AIMS Math., **4**(2019), No. 6, 1554-1568.
- [19] M.A. Noor, K.I. Noor and M.T. Rassias: *New trends in general variational inequalities*, Acta Appl. Math., **170**(2020), 981-1064.
- [20] J. Pečarić, F. Proschan, Y.L. Tong: *Convex Functions, Partial Orderings and Statistical Applications*, Academic Press, New York, 1992 (MSE 187).
- [21] J. Pečarić, C.E.M. Pearce and V. Šimić: *Stolarsky means and Hadamard's inequality*, J. Math. Anal. Appl., **220**(1998), 99-109.
- [22] J. Jakšetić and J. Pečarić: *On exponential convexity, Euler-Radau expansions and stolarsky means*, Rad Hrvat. Akad. Znan. Umjet. Mat. Znan., **515**(2013), 81-94.
- [23] S. Rashid, M.A. Noor and K.I. Noor: *Fractional exponentially m -convex functions and inequalities*, Inter. J. Anal. Appl., **17**(2019), No. 3, 464-478.
- [24] 19B S. Rashid, M.A. Noor, K.I. Noor and A.O. Akdemir: *Some new generalizations for exponentially s -convex functions and inequalities via fractional operators*, Fractal Fract., **3**(2019), Art. No. 24, 16 pages.
- [25] S. Pal and T.K. Wong: *On exponentially concave functions and a new information geometry*, Ann. Probab., **46**(2018), No. 2, 1070-1113.
- [26] Y.X. Zhao, S.Y. Wang and L. Coladas Uria: *Characterizations of r -convex functions*, J. Optim. Theory Appl., **145**(2010), 186-195.

University of Wah

Department of Mathematics

Wah Cantt, Pakistan

Email address: khalidan@gmail.com, inayat.noor@uow.edu.pk

University of Wah

Department of Mathematics

Wah Cantt, Pakistan

Email address: noormaslam@gmail.com, aslam.noor@uow.edu.pk