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EXACT NULL CONTROLLABILITY OF SEMILINEAR NONLOCAL AND NON-AUTONOMOUS SYSTEM

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Abstract. In this article, by using the theory of linear evolution system and Schauder fixed point theorem, we study the existence of mild-solutions for fractional semilinear systems of non-autonomous deformable with nonlocal conditions in Hilbert spaces. Furthermore, we establish the exact null controllabilty of these systems. An example is provided to show the practical application of our results.

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