

**EXPONENTIAL STABILITY OF A DAMPED TIMOSHENKO BEAM
WITH SPATIALLY VARYING PHYSICAL PARAMETERS**

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ABSTRACT. This paper investigates the Riesz basis property and exponential stability of a damped Timoshenko beam with spatially varying physical parameters. The beam is clamped at one end and subject to dynamic boundary feedback at the free end. Reformulating the model as an abstract Cauchy problem in a weighted Hilbert space, we establish well-posedness and energy decay through semigroup theory. Using Birkhoff-Langer-type asymptotic spectral analysis, we prove that the system is Riesz-spectral and derive sharp exponential stability conditions. These conditions explicitly reveal the influence of the boundary controls through logarithmic terms in the eigenvalue distribution, even in the presence of sign-changing internal damping.

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