

MORPHISMS AND ALGEBRAIC POINTS ON THE CURVES

$$C_{r,s}(7) : y^7 = x^r(x-1)^s$$

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ABSTRACT. The purpose of this paper is to determine the morphisms and algebraic points of degree 3 on the Fermat quotients $C_{r,s}(7) : y^7 = x^r(x-1)^s$, where r and s are integers such that $1 < r, s, r + s < 7$. We explicitly determine the birational morphisms between the curves $C_{1,2}(7)$ and $C_{2,1}(7)$ as well as $C_{1,2}(7)$ and $C_{4,1}(7)$, and then the set of algebraic points of degree 3 on the curves $C_{2,1}(7)$ and $C_{4,1}(7)$.

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