

CHARACTERIZATION OF NORMED INTERVAL SPACE WITHOUT USING SPACE COMPLETENESS

SUMIT PANT AND NARENDRA KUMAR SINGH

ABSTRACT. Space completeness has been a key factor in all prior investigations of near fixed point theorems in the context of ‘Normed Interval Space’ (*NIS*). The study utilizes the Kannan contraction condition as a framework to introduce two novel findings in the field of *NIS*. The first theorem assumes that the space is complete, while the second theorem does not make this assumption. In addition, we have provided examples that clearly show the distinction between Wu’s previous near fixed point theorem, which relies on Banach contraction, and the theorems we have proposed. In the latter part, the concept of an interval circle is introduced, along with the notion of equivalence class of interval circles and the near fixed interval circle. Furthermore, a novel theorem regarding the near fixed interval circle for *NIS* is derived and extensively discussed.

REFERENCES

- [1] S. Banach: *Sur les opérations dans les ensembles abstraits et leur application aux équations intégrales*, Fund. Math., **3**(1922), No. 1, 133-181.
- [2] K. Hoffman and R. Kunze: *Linear Algebra*, Prentice-Hall, New Jersey, 1971.
- [3] R. Kannan: *Some results on fixed points*, Bull. Calcutta Math. Soc., **60**(1968), 71-76.
- [4] R. Kannan: *Some results on fixed points II*, Amer. Math. Monthly, **76**(1969), No. 4, 405-408.
- [5] E. Kreyszig: *Introductory Functional Analysis with Applications*, Wiley, New York, 1991.
- [6] N. Mlaiki, U. Çelik, N. Taş, N.Y. Özgür and A. Mukheimer: *Wardowski type contractions and the fixed-circle problem on S-metric spaces*, J. Math., **2018**(2018), Article ID 9127486, 9 pages.
- [7] N. Mlaiki, N. Taş and N.Y. Özgür: *On the fixed-circle problem and Khan type contractions*, Axioms, **7**(2018), No. 4, Art. No. 80, 10 pages.
- [8] N. Özgür: *Fixed-disc results via simulation functions*, Turkish J. Math., **43**(2019), No. 6, 2794-2805.
- [9] N.Y. Özgür and N. Taş: *Some fixed-circle theorems on metric spaces*, Bull. Malays. Math. Sci. Soc., **42**(2019), 1433-1449.
- [10] N.Y. Özgür, N. Taş and U. Çelik: *New fixed-circle results on S-metric spaces*, Bull. Math. Anal. Appl., **9**(2017), No. 2, 10-23.
- [11] N. Taş: *Suzuki-Berinde type fixed-point and fixed-circle results on S-metric spaces*, J. Linear Topol. Algebra, **7**(2018), No. 3, 233-244.
- [12] N. Taş, N.Y. Özgür and N. Mlaiki: *New types of F_c -contractions and the fixed-circle problem*, Mathematics, **6**(2018), No. 10, Art. No. 188, 9 pages.
- [13] H.C. Wu: *A new concept of fixed point in metric and normed interval spaces*, Mathematics, **6**(2018), No. 11, Art. No. 219, 23 pages.
- [14] H.C. Wu: *Normed interval space and its topological structure*, Mathematics, **7**(2019), No. 10, Art. No. 983, 22 pages.
- [15] H.C. Wu: *Near fixed point theorems in near Banach spaces*, AIMS Math., **8**(2023), No. 1, 1269-1303.

Received: February 25, 2024. *Revised:* July 08, 2024.

2020 Mathematics Subject Classification: 47H10, 54H25.

Key words and phrases: *NIS*, near fixed point, norm contraction, equivalence class, near fixed interval circle.

G.N.A.P.G. College
Department of Mathematics
Bhatapara-493118, Baloda Bazar, Chhattisgarh, India
E-mail address: sumit4maths@gmail.com

M.B. Govt. P.G. College
Department of Mathematics
Haldwani-263139, Nainital, Uttarakhand, India
E-mail address: nsijwali@gmail.com