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SOME NOVEL ASPECTS AND APPLICATIONS OF NOOR ITERATIONS AND NOOR ORBITS

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Abstract. In this paper, we give a brief idea of Noor iterations, which were introduced by Noor in 2000 and had influenced several areas of applicable sciences including fractal geometry, chaos, design, financial, population, climate, language. Noor iterations and Noor orbits are related to fixed point theory, logistic maps, machine learning, artificial intelligence, Julia sets, Mandelbot sets. These iterations perform better than the Mann iteration (1953), Ishikawa iterations (1974) and being used in the design of solar panels related to green innovation (climate related problems). In recent years, Noor iterations have been modified and generalization in several directions using novel and innovative techniques to consider complicated complex problems.

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