

ON PSEUDO-RIEMANNIAN $\mathcal{A}$ -MANIFOLDS

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ABSTRACT. An $\mathcal{A}$ -manifold is a Riemannian manifold whose Ricci tensor is cyclic parallel. This concept generalizes the Einstein manifolds. In this article, we extend the notion of Riemann $\mathcal{A}$ -manifold to the notion of pseudo-Riemannian $\mathcal{A}$ -manifold. As applications, we construct non-trivial examples of pseudo-Riemannian $\mathcal{A}$ -manifolds on the cotangent bundle of an affine surface using the deformed Riemannian extension.

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