

GRAY'S DECOMPOSITION ON WARPED PRODUCT MANIFOLDS

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*The paper is dedicated to the memory of Professor Momo Bangoura who passed away 27 May 2023.
 We are grateful toward him for sharing his knowledges and experiences with us.*

ABSTRACT. In [11], Gray introduced two interesting classes of Riemannian manifolds which generalize the concept of Einstein manifolds. In this paper, we prove that the projection of the Einstein-like class type defined on a warped product induces Einstein-like type on the base manifolds whereas the fiber manifold does under some conditions.

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