

SOME ASPECTS OF EXPONENTIALLY GENERAL NONCONVEX VARIATIONAL INEQUALITIES

MUHAMMAD ASLAM NOOR AND KHALIDA INAYAT NOOR

ABSTRACT. In this paper, some new classes of exponentially general nonconvex variational inequalities are introduced and investigated. Several special cases are discussed as applications of these nonconvex variational inequalities. Projection technique is used to establish the equivalence between the nonconvex variational inequalities and fixed point problem. This equivalent formulation is used to discuss the existence of the solution. Several inertial type methods are suggested and analyzed for solving exponentially general nonconvex variational inequalities. using the technique of the projection operator and dynamical systems. Convergence analysis of the iterative methods is analyzed under suitable and appropriate weak conditions. In this sense, our result can be viewed as improvement and refinement of the previously known results. Our methods of proof are very simple as compared with other techniques.

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Muhammad Aslam Noor
 Mathematics, COMSATS University Islamabad
 Islamabad, Pakistan
 E-mail address: noormaslam@gmail.com

Khalida Inayat Noor
 Mathematics, COMSATS University Islamabad
 Islamabad, Pakistan
 E-mail address: khalidan@gmail.com