

LARGE DEVIATIONS AND HOMOGENIZATION OF SEMILINEAR NONLOCAL PDE WITH THE NEUMANN BOUNDARY CONDITION

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ABSTRACT. According to the Neumann boundary condition, we consider some family of nonlocal operators related to reflected stochastic differential equations driven by multiplicative isotropic α -stable Lévy noise ($1 < \alpha < 2$) in a domain $D \subset \mathbb{R}^d$. We study by using homogenization theory the behavior of $u^{\varepsilon, \delta} : D \rightarrow \mathbb{R}$ of semilinear partial differential equations with periodic coefficients varying over length scale δ and nonlinear reaction term of scale $1/\varepsilon$. The behavior is required as ε, δ both tend to 0. Our homogenization method is entirely probabilistic. Since δ decreases faster than ε , we may apply the large deviations principle with homogenized coefficients.

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